MA 161 - Lesson 9 (3.1)

Today: Introduction to Derivative — NOT in exam 1
Slope of tangent line
Equation of tangent line

Office Hours! This Week Monday, Wednesday 2:45pm - 4:15pm, After Class
Thursday 2:30pm - 3:30pm.

Announcements: Exam 1 on Thursday 09/18, 8pm - 9pm
↳ Study Guide, instructions, Seating Chart on Brightspace.

HW 8,9 Due: Tuesday 09/16

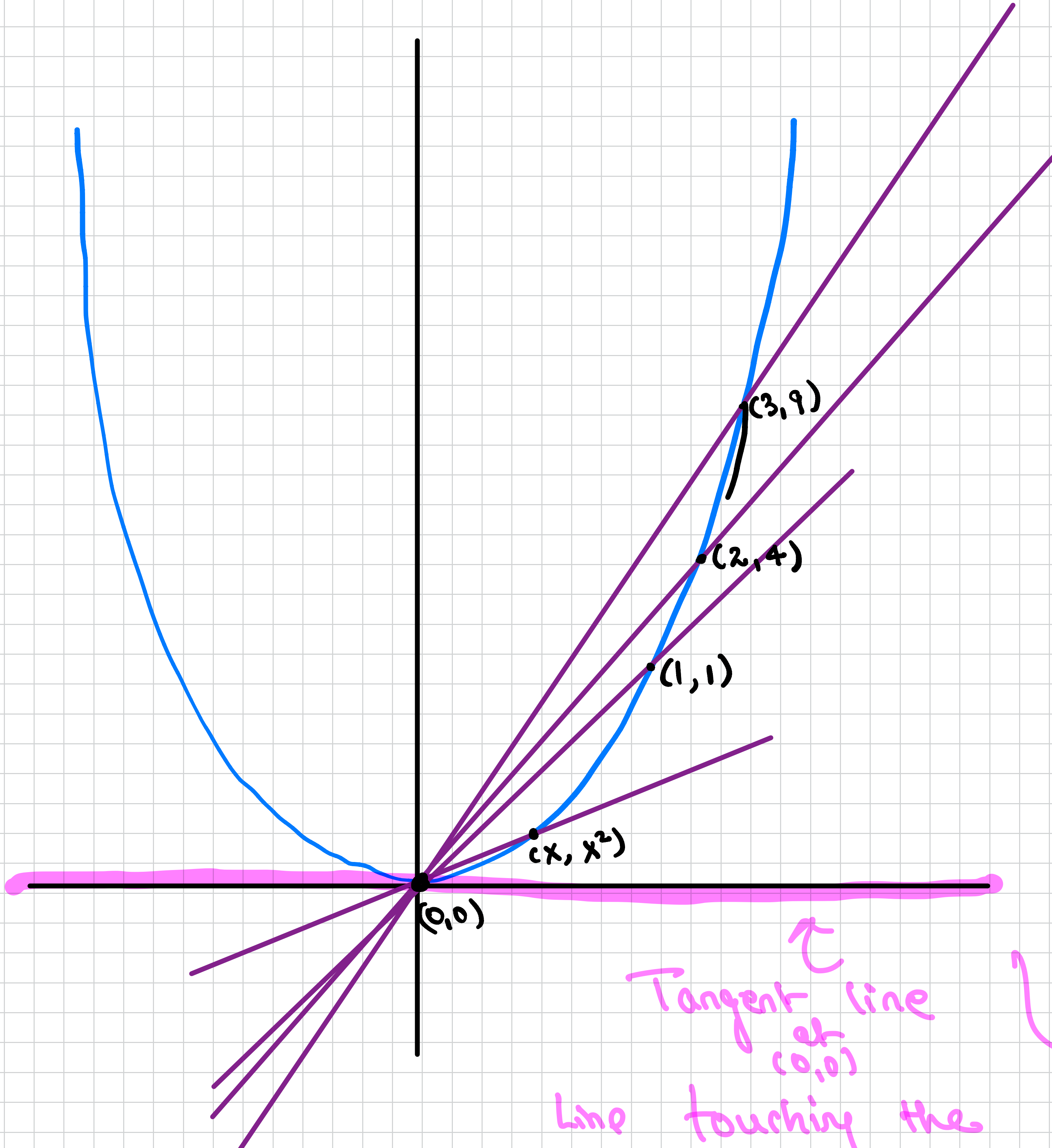
Quiz 5 (Lesson 7): Thursday 09/16

Feasting with Faculty

Warmup: Slope of Secant Lines

Secant Lines: line through 2 points!

Let $f(x) = x^2$



Tangent line at (0,0)

Line touching the parabola at (0,0)

Here we have Horizontal line $\Rightarrow$ slope $= 0$

Slope of Secant Lines:

through $(0,0)$, $(3,9)$ $= \dfrac{9-0}{3-0} = 3$

through $(0,0)$, $(2,4)$ $= \dfrac{4-0}{2-0} = 2$

through $(0,0)$, $(1,1)$ $= 1$

through $(0,0)$, (x, x^2) $= \dfrac{x^2-0}{x-0} = x.$

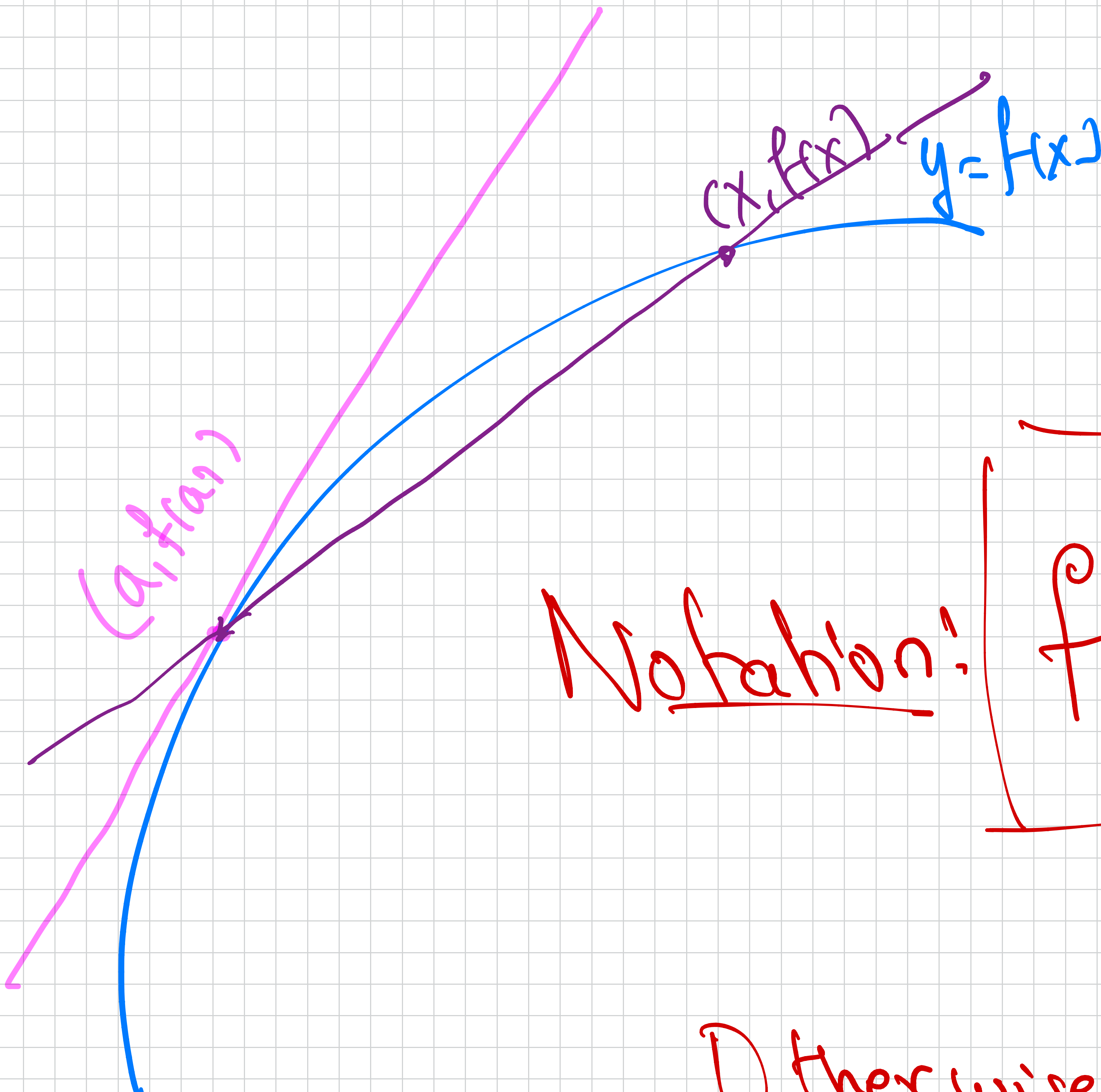
Slope of tangent line = limit of slope of secant lines as $x \to 0$

$= \lim_{x \to 0} x = 0.$

Derivative of a function $f(x)$ at $x = 0$ $\Big\}$ = slope of tangent line at $x = a$

= limit of the slope of secant line

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



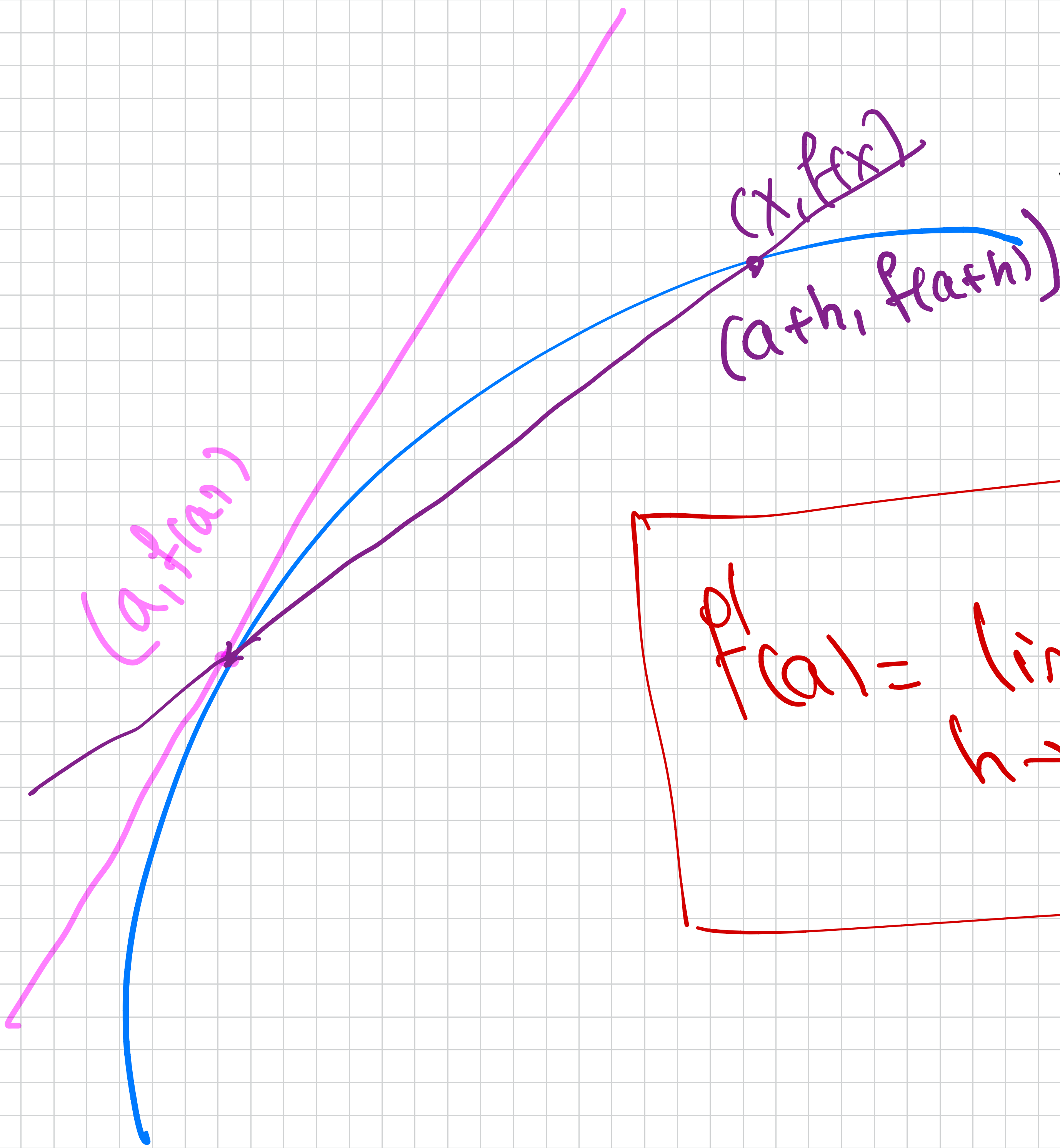
Notation: $\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$

if limit exists & if finite

Otherwise we say derivate doesnot exist at $x = a$.

Another Formula for Derivative:

instead of secant line through $(a, f(a))$, $(x, f(x))$, $x \to a$



think of it as secant line through $(a, f(a))$, $(a+h, f(a+h))$ $h \to 0$

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

eg: $f(x) = x^2$, find $f'(2)$

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

factor: $(x-2)(x+2)$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2} x + 2$$

$$= 4.$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$$

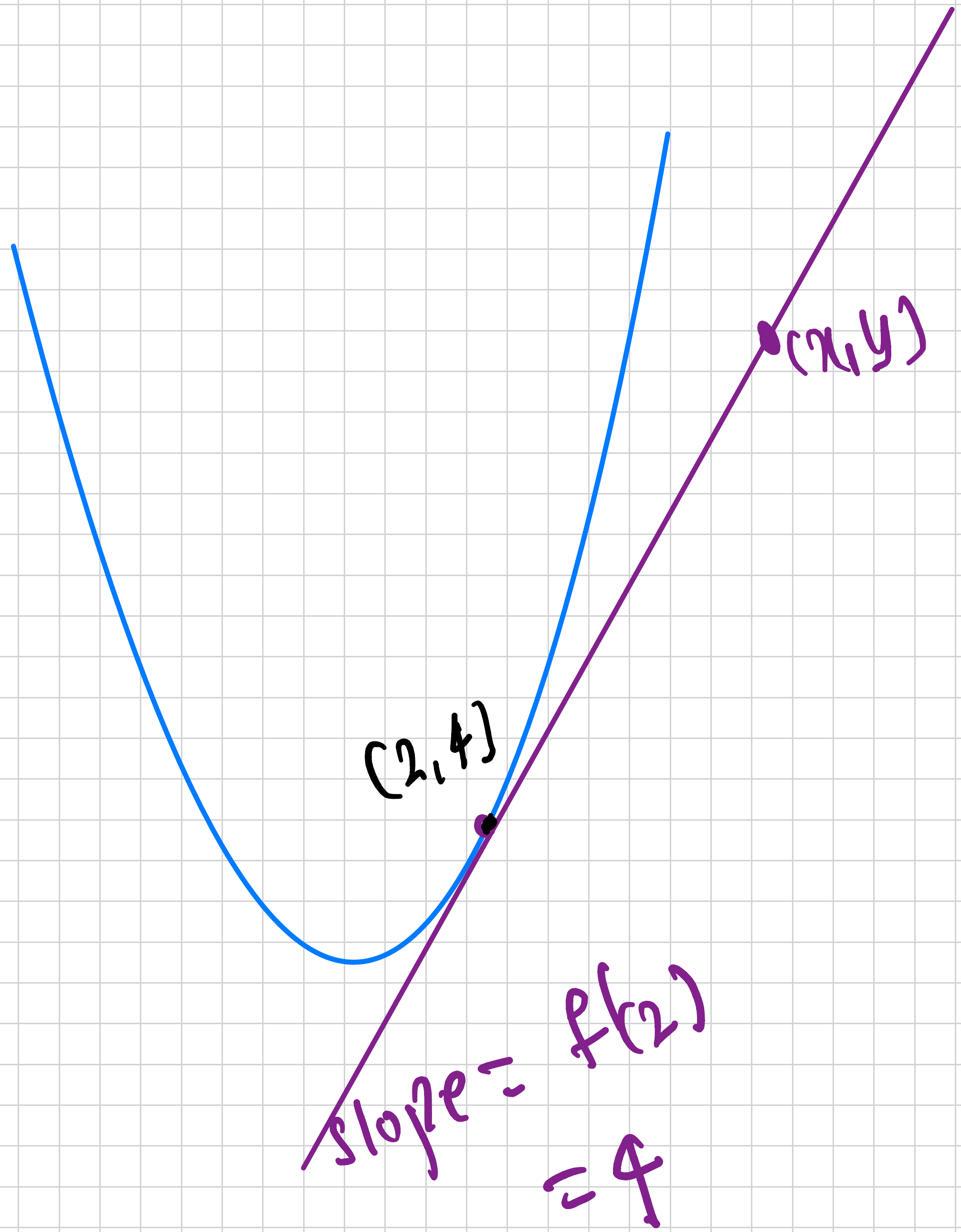
$$= \lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4 + h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 4 + h$$

$$= 4.$$

eg: Find equation of tangent to $y = f(x) = x^2$ at $(2,4)$



Equation of a line through (x_0, y_0) with slope m — Point slope form

$$\frac{y - y_0}{x - x_0} = m$$

$$\frac{y-4}{x-2} = 4$$

$$y = 4(x-2) + 4$$

$$\boxed{y = 4x - 4}$$

Finding formula for Derivative of $f(x) = x^2$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{x^2 - a^2}{x - a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x+a)}{\cancel{x-a}}$$

$$= \lim_{x \to a} x + a = 2a$$

$f'(a) = 2a$, for any a

e.g.: $f(x) = \dfrac{1}{1+7x}$, evaluate $f'(3)$

$f(3) = \dfrac{1}{22}$

$$\lim_{x \to 3} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3} \frac{\frac{1}{1+7x} - \frac{1}{22}}{x-3}$$

idea: make it one fraction

$$= \lim_{x \to 3} \frac{\frac{(22) - (1+7x)}{(22)(1+7x)}}{(x-3)}$$

$21 - 7x$
$= -7(x-3)$

$$= \lim_{x \to 3} \frac{-7\cancel{(x-3)}}{22(1+7x)\cancel{(x-3)}}$$

$$= \frac{-7}{(22)^2}$$

$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{1+7(3+h)} - \frac{1}{22}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{22+7h} - \frac{1}{22}}{h}$$

make it one fraction

$$= \lim_{h \to 0} \frac{\cancel{22} - (\cancel{22} + 7\cancel{h})}{\cancel{h}(22+7h)(22)}$$

$$= \lim_{h \to 0} \frac{-7}{(22+7h)(22)} = \frac{-7}{(22)^2}$$

eg: $f(x) = \sqrt{3x+4}$, find $f'(4)$

$$f'(4) = \lim_{x\to 4} \frac{f(x) - f(4)}{x-4}$$

$$= \lim_{x\to 4} \frac{\sqrt{3x+4} - 4}{x-4}$$

$3x+4-16$
$= 3x-12$
$= 3(x-4)$

$$= \lim_{x\to 4} \frac{(\sqrt{3x+4} - 4)(\sqrt{3x+4} + 4)}{(x-4)(\sqrt{3x+4} + 4)}$$

$$= \lim_{x\to 4} \frac{3\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{3x+4} + 4)} = \frac{3}{\sqrt{16} + 4}$$

$$= \frac{3}{8}.$$

eg: Suppose $f'(a) = \lim_{h \to 0} \frac{(5+h)^3 - 5^3}{h}$

Guess the function $f(x)$ and the value a.

II formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Guess: $f(x) = x^3$

$a = 5$

Compute $f'(5)$!

$$f'(5) = \lim_{h \to 0} \frac{(5+h)^3 - 5^3}{h} = \lim_{h \to 0} \frac{\cancel{125} + 75h + 15h^2 + h^3 \cancel{-125}}{h}$$

$= (5+h)(5+h)(5+h)$

$= 125 + 75h + 15h^2 + h^3$

$$= \lim_{h \to 0} 75 + 15h + h^2$$

$$= \underline{\underline{75}}.$$